Type-I vacua and brane transmutation

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Abstract

In a classic paper, Fradkin and Tseytlin showed how magnetic deformations can be introduced in open strings. In this contribution we review some recent work on type-I vacua with magnetised branes and describe the role of additional discrete deformations, related to quantised values of the NS-NS antisymmetric tensor B_{ab}.

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dedicated to the memory of Professor Efim S. Fradkin

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1. Introduction

We are pleased and honoured to contribute to this celebration of Prof. Efim S. Fradkin. His work has had a profound influence on String Theory, and in particular on the recent results that we have chosen to review here. In addition, one of us (A.S.) had the privilege of meeting Prof. Fradkin on several occasions, where anyone could enjoy his friendly attitude in sharing his insights, while witnessing his pervasive enthusiasm for scientific research.

Our starting point is the classic paper [1], where Fradkin and Tseytlin showed how to introduce magnetic deformations in open strings and linked the Born-Infeld action to String Theory. These results, supplemented by the elegant canonical analysis of [2], are central to most current developments related to D-brane physics and non-commutative geometry. Our interest in the magnetic deformations of [1], however, is not directly related to non-commutative geometry. Rather, it can be traced to the proposal of Bachas [3] of attaining the breaking of space-time supersymmetry via string compactifications on magnetised tori. This setting can be regarded as a string realisation of the early field theory work of Witten [4] and, as in that case, was actually restricted to configurations with vanishing instanton density.

In a recent work [5], we relaxed the restriction to vanishing instanton density, and showed how the brane content of toroidal and orbifold models can be affected in an interesting way. Magnetised D9 branes can then acquire a D5 charge, and special configurations with self-dual internal fields can even result in new interesting vacua with unbroken *supersymmetry*. Whereas the phenomenon of brane transmutation, a consequence of the peculiar Wess-Zumino coupling of D-branes, was previously discussed in [6], together with the restrictions associated with unbroken supersymmetry [7], the models presented in [5] and reviewed here are the first consistent vacuum configurations for type-I strings where this setting is realised. They stand out for their gauge groups with peculiar ranks and for the structure of their matter representations, that occur in multiple families. This construction actually requires a small additional step beyond [3], the extension of magnetic deformations to orbifold models. The reason for this is somewhat technical: a supersymmetric configuration requires that only D9 and D5 branes be present, as opposed to their antibranes, and only orbifolds, as opposed to tori, can contain the O5 planes capable of absorbing the D5 charge of magnetised D9 branes. The resulting constructions may be regarded as explicit string realisations of the inverse of the process originally advocated in [8], where a five-brane is recovered from small instantons: reverting this process one can associate conventional fat instantons to magnetised D9 branes[2].

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[2]We are grateful to A. Uranga for a discussion on this point.

We shall conclude with some new results on the effect of a quantised NS-NS field B_{ab} on these magnetised branes. As usual, this field is projected out in conventional type-I vacua [9] that, however, allow interesting discrete deformations associated with its discrete values. In tori [10], these result in gauge groups of reduced rank and allow one to interpolate between the orthogonal and symplectic cases. In orbifolds [11, 12], these effects are accompanied by a multiplet structure for the fixed points, that results in the appearance of several tensor multiplets in the projected closed sector and of multiple families of 95 states, as originally noted in the rational construction of [13]. Although we shall confine our attention to the D9-D5 case, T-duality can be used to relate this setting to a number of similar ones involving other types of branes. This applies, in particular, to the discrete deformations associated to B_{ab}, that can be related to discrete choices for the geometry of the brane configuration, as in [14].

2.   Magnetised orbifolds and brane transmutation

Let us begin by reviewing briefly the results of [5]. Some intuitive field theory arguments suffice to expose the essence of the phenomenon, and are well captured by the low-energy effective action for D9 branes in an internal Abelian background [3],

$$\mathcal{S}_9 = -T_{(9)} \int_{\mathcal{M}_{10}} \mathrm{d}^{10}x \; e^{-\phi} \sum_{a=1}^{32} \sqrt{-\det\left(g_{10} + q_a F\right)} - \mu_{(9)} \sum_{p,a} \int_{\mathcal{M}_{10}} e^{q_a F} \wedge C_{p+1} + \dots \quad , \quad (1)$$

where a labels the types of Chan-Paton (CP) charges that couple to the magnetic fields with strength q_a, and

$$T_{(p)} = \sqrt{\frac{\pi}{2\kappa^2}} \left(2\pi\sqrt{\alpha'}\right)^{3-p} = |\mu_{(p)}| \; . \quad (2)$$

Here T and μ are the tension and the R-R charge for a type-I Dp brane [15], while κ defines the ten-dimensional Newton constant $G_N^{(10)} = \kappa^2/8\pi$. To illustrate the phenomenon, it suffices to consider the geometry $\mathcal{M}_{10} = \mathcal{M}_6 \times T^2 \times T^2$ with constant Abelian magnetic fields H_1 and H_2 lying in the two internal tori. These are monopole fields, and thus satisfy the Dirac quantisation conditions

$$q\, H_i\, v_i = k_i \qquad (i = 1, 2) \; , \quad (3)$$

where, aside from powers of 2π, $v_i = R_i^{(1)} R_i^{(2)}/\alpha'$ are the dimensionless volumes of the two tori of radii $R_i^{(1)}$ and $R_i^{(2)}$, k_i are the degeneracies of the corresponding Landau levels and q is the elementary electric charge for the system. As anticipated, we forego the restriction in [4, 3] and actually pick a pair of Abelian fields aligned with the same U(1)

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[3]As in [5], the (dimensionless) magnetic fields differ from the conventional ones by a $2\pi\alpha'$ rescaling.

subgroup. This affects the Wess-Zumino coupling, giving rise to an effective D5 charge, so that

$$
\begin{aligned}
\mathcal{S}_9 \;=\; & -\,T_{(9)} \int_{\mathcal{M}_{10}} \mathrm{d}^{10}x \; e^{-\phi} \; \sqrt{-g_6} \; \sum_{a=1}^{32} \sqrt{(1+q_a^2 H_1^2)(1+q_a^2 H_2^2)} \\
& -\, 32\,\mu_{(9)} \int_{\mathcal{M}_{10}} C_{10} \;-\; \left(2\pi\sqrt{\alpha'}\right)^4 \mu_{(9)} \; v_1 v_2 \; H_1 \, H_2 \sum_{a=1}^{32} q_a^2 \int_{\mathcal{M}_6} C_6 \;,
\end{aligned}
\tag{4}
$$

where g_6 denotes the six-dimensional space-time metric, and where for simplicity we have chosen an identity metric in the internal space. In particular, if the two internal fields have identical magnitudes, for the resulting (anti)self-dual configuration the action becomes

$$
\begin{aligned}
\mathcal{S}_9 \;=\; & -\, 32 \int_{\mathcal{M}_{10}} \Big(\mathrm{d}^{10}x \; \sqrt{-g_6} \; T_{(9)} \; e^{-\phi} + \mu_{(9)} \, C_{10} \Big) \\
& -\, \sum_{a=1}^{32} \left(\frac{q_a}{q}\right)^2 \int_{\mathcal{M}_6} \Big(\mathrm{d}^6x \; \sqrt{-g_6} \; |k_1 k_2| \; T_{(5)} \; e^{-\phi} + k_1 k_2 \, \mu_{(5)} \, C_6 \Big) \;.
\end{aligned}
\tag{5}
$$

Therefore, not only the Dirac quantisation conditions (3) have compensated the integration over the internal tori, but in the second line of (5) the additional powers of α' have nicely converted $T_{(9)}$ and $\mu_{(9)}$ into $T_{(5)}$ and $\mu_{(5)}$. As a result, a D9 brane on a magnetised $T^2 \times T^2$ indeed mimics a D5 brane or a D5 antibrane according to whether the orientations of H_1 and H_2, reflected by the relative sign of k_1 and k_2, are identical or opposite.

We can now extend the analysis to String Theory, following [9]. A precise control of the CFT exhibits very nicely several properties of the effective action, including new couplings to twisted states of orbifold models. As we anticipated, supersymmetry is generically broken, but supersymmetric vacuum configurations can be obtained starting from an orbifold that normally requires the introduction of D5 branes. The simplest such instance is the six-dimensional compactification on $(T^2 \times T^2)/Z_2$ with Klein-bottle projection

$$
\mathcal{K} = \tfrac{1}{4} \left\{ (Q_o + Q_v)(0;0) \, [P_1 P_2 + W_1 W_2] + 16 \times 2 (Q_s + Q_c)(0;0) \left(\frac{\eta}{\vartheta_4(0)}\right)^2 \right\} \;,
\tag{6}
$$

that corresponds to the introduction of $O9_+$ and $O5_+$ planes, and thus to a projected $\mathcal{N} = (1,0)$ supersymmetric closed spectrum with one tensor multiplet and 20 hypermultiplets. For the sake of brevity, in this Section we shall confine our attention to the models of [5] without D5 branes, where the O5 charge is fully compensated by the magnetised D9 branes. The characters used in eq. (6) and in the following are in general combinations of theta-functions with non-vanishing arguments, that extend the standard $(1,0)$ supersymmetric combinations, and are described in detail in [5].

As in our field theory considerations, we introduce a pair of internal magnetic fields aligned with the same U(1) subgroup of SO(32), and we restrict our attention to the

maximal residual gauge group, $U(m) \times U(n)$, with $m + n = 16$. In writing the direct-channel annulus amplitude, let us begin by recalling [2] that a uniform magnetic field with components H_1 and H_2 in the two internal tori alters the boundary conditions for open strings, shifting their mode frequencies by

$$z_i^{\text{L,R}} = \frac{1}{\pi} \left[\tan^{-1}(q_{\text{L}} H_i) + \tan^{-1}(q_{\text{R}} H_i) \right] , \qquad (7)$$

where q_{L} (q_{R}) denotes the charge of the left (right) end of the open string with respect to the $U(1)$ field H_i. A further novelty [2] is displayed by "dipole" strings, with opposite end charges, whose oscillator modes are unaffected, but whose world-sheet coordinates undergo a complex "boost", so that their Kaluza-Klein momenta m_i are rescaled according to

$$m_i \rightarrow \frac{m_i}{\sqrt{1 + q_a^2 H_i^2}} . \qquad (8)$$

The direct-channel annulus amplitude is

$$
\begin{aligned}
\mathcal{A} = \frac{1}{4} \Bigg\{ & (Q_o + Q_v)(0;0) \left[(m + \bar{m})^2 P_1 P_2 + 2 n \bar{n} \tilde{P}_1 \tilde{P}_2 \right] \\
& - 2(m + \bar{m})(n + \bar{n})(Q_o + Q_v)(z_1 \tau; z_2 \tau) \frac{k_1 \eta}{\vartheta_1(z_1 \tau)} \frac{k_2 \eta}{\vartheta_1(z_2 \tau)} \\
& - (n^2 + \bar{n}^2)(Q_o + Q_v)(2z_1 \tau; 2z_2 \tau) \frac{2 k_1 \eta}{\vartheta_1(2z_1 \tau)} \frac{2 k_2 \eta}{\vartheta_1(2z_2 \tau)} \\
& - \left[(m - \bar{m})^2 - 2 n \bar{n} \right] (Q_o - Q_v)(0;0) \left(\frac{2\eta}{\vartheta_2(0)} \right)^2 \\
& - 2(m - \bar{m})(n - \bar{n})(Q_o - Q_v)(z_1 \tau; z_2 \tau) \frac{2\eta}{\vartheta_2(z_1 \tau)} \frac{2\eta}{\vartheta_2(z_2 \tau)} \\
& - (n^2 + \bar{n}^2)(Q_o - Q_v)(2z_1 \tau; 2z_2 \tau) \frac{2\eta}{\vartheta_2(2z_1 \tau)} \frac{2\eta}{\vartheta_2(2z_2 \tau)} \Bigg\} , \qquad (9)
\end{aligned}
$$

while the corresponding Möbius amplitude is

$$
\begin{aligned}
\mathcal{M} = -\frac{1}{4} \Bigg\{ & (\hat{Q}_o + \hat{Q}_v)(0;0) \left[(m + \bar{m}) P_1 P_2 \right] \\
& - (n + \bar{n})(\hat{Q}_o + \hat{Q}_v)(2z_1 \tau; 2z_2 \tau) \frac{2 k_1 \hat{\eta}}{\hat{\vartheta}_1(2z_1 \tau)} \frac{2 k_2 \hat{\eta}}{\hat{\vartheta}_1(2z_2 \tau)} \\
& - (m + \bar{m}) (\hat{Q}_o - \hat{Q}_v)(0;0) \left(\frac{2\hat{\eta}}{\hat{\vartheta}_2(0)} \right)^2 \\
& - (n + \bar{n})(\hat{Q}_o - \hat{Q}_v)(2z_1 \tau; 2z_2 \tau) \frac{2\hat{\eta}}{\hat{\vartheta}_2(2z_1 \tau)} \frac{2\hat{\eta}}{\hat{\vartheta}_2(2z_2 \tau)} \Bigg\} . \qquad (10)
\end{aligned}
$$

The arguments z_i ($2z_i$) are associated to strings with one (two) charged ends while, for the sake of brevity, both the imaginary modulus $\frac{1}{2} it$ of $\mathcal{A}$ and the complex modulus $\frac{1}{2} + \frac{1}{2} it$ of

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$\mathcal{M}$ are denoted by the same symbol τ, although the proper "hatted" contributions to the Möbius amplitude are explicitly indicated. P_i and W_i are conventional momentum and winding sums for the two-tori, while a "tilde" denotes a sum with momenta "boosted" as in (8). Finally, m and n (together with their conjugates $\bar{m}$ and $\bar{n}$) are CP multiplicities for the D9 brane, and several terms with with opposite z_i arguments have been grouped together, using the symmetries of the Jacobi theta-functions.

For generic magnetic fields, the open spectrum is indeed non-supersymmetric and develops Nielsen-Olesen instabilities [16]. As emphasised in [3], the emergence of these tachyonic modes can be ascribed to the magnetic couplings of the internal components of gauge fields. For instance, small magnetic fields affect the mass formula for the untwisted string modes according to

$$\Delta M^2 \;=\; \frac{1}{2\pi\alpha'} \sum_{i=1,2} \left[(2n_i+1)|(q_{\mathrm{L}}+q_{\mathrm{R}})H_i| + 2(q_{\mathrm{L}}+q_{\mathrm{R}})\Sigma_i H_i\right] \;, \tag{11}$$

where the first term originates from the Landau levels and the second from the magnetic moments of the spins Σ_i. For the internal components of the vectors, the magnetic moment coupling generally overrides the zero-point contribution, leading to tachyonic modes, unless $|H_1| = |H_2|$, while for spin-$\frac{1}{2}$ modes it can at most compensate it. Moreover, if $H_1 = H_2$ the supersymmetry charge, that belongs to C_4C_4, is also unaffected. On the other hand, if $H_1 = -H_2$ one obtains models with "brane supersymmetry breaking", similar in spirit to those of [17, 12, 18, 19, 20]. However, in this case supersymmetry is broken on the whole magnetised D9 brane, since we are working effectively in the presence of blown-up instantons.

The untwisted R-R tadpole conditions arising from the $C_4S_2C_2$ sector read

$$\left[m + \bar{m} + n + \bar{n} - 32 + q^2 H_1 H_2(n+\bar{n})\right]\sqrt{v_1 v_2} - \frac{32}{\sqrt{v_1 v_2}} = 0 \;, \tag{12}$$

aside from terms that vanish after identifying the multiplicities of conjugate representations $(m, \bar{m})$ and $(n, \bar{n})$. The additional (untwisted) R-R tadpole conditions from Q_o and Q_v are compatible with (12) and do not add further constraints. This expression reflects the familiar Wess-Zumino coupling of eq. (1), and therefore the various powers of H correspond to R-R forms of different degrees. In particular, as we anticipated in our field theory discussion, the term bilinear in the magnetic fields has a very neat effect: it charges the D9 brane with respect to the six-form potential. This can be seen very clearly making use of the quantisation condition (3), that turns the tadpole conditions (12) into

$$\begin{aligned} m + \bar{m} + n + \bar{n} &= 32 \;, \\ k_1 k_2 (n + \bar{n}) &= 32 \;. \end{aligned} \tag{13}$$

In a similar fashion, the untwisted NS-NS tadpoles exhibit very nicely their relation to the Born-Infeld term in (1), and can be linked to its derivatives with respect to the corresponding moduli, while the twisted NS-NS tadpoles display new couplings to twisted states present in the effective Lagrangian. A novelty is that some of the tadpoles are *not* perfect squares, as a result of the peculiar behaviour of the internal magnetic deformations under world-sheet time reversal. All these features are described in some detail in [5]. While for generic internal magnetic fields it is impossible to satisfy the NS-NS tadpoles, the supersymmetric choice $H_1 = H_2$ makes them all nicely compatible with the R-R ones.

We can now describe the low-lying spectrum of a model without D5 branes and with $k_1 = k_2 = 2$, the minimal Landau-level degeneracies that on this Z_2 orbifold are compatible with the positivity of the direct channel. Although the closed spectrum is the standard one, and comprises the $\mathcal{N} = (1, 0)$ gravitational multiplet, together with one tensor multiplet and twenty hypermultiplets, the open spectrum is quite different from the familiar one of [13, 21], with gauge group $U(16)|_9 \times U(16)|_5$. Having excluded the D5 branes, the solution of the tadpole conditions yields $m = 12$, $n = 4$, and the result is a rather unusual supersymmetric Z_2 model, with a gauge group of rank 16, $U(12) \times U(4)$, and with charged hypermultiplets in the representations $(66 + \overline{66}, 1)$, in five copies of the $(1, 6 + \overline{6})$, and in four copies of the $(\overline{12}, 4)$. A distinctive feature of this spectrum, that is free of all irreducible gauge and gravitational anomalies, consistently with the vanishing of all RR tadpole conditions [22], as well as of the similar ones in [5], is that some of the matter occurs in multiple families. This peculiar phenomenon is a consequence of the multiplicities of Landau levels, that in these Z_2 orbifolds with vanishing B_{ab} are multiples of two for each magnetised torus. Notice that, when D5 branes are also present [5], one is led in general to rank reductions, but not simply by powers of two as in the presence of a quantised B_{ab} [10, 11, 12]. These are not the first concrete examples of brane transmutation in type I vacua but, to the best of our knowledge, they are the first supersymmetric ones. Indeed, Z_2 orientifolds without D5 branes appeared previously in [23], where magnetised fractional D9 branes were used to build six-dimensional asymmetric orientifolds with "brane supersymmetry breaking".

3. Introducing a quantised B_{ab}

In this Section we extend the construction of [5] to allow for quantised values of the NS-NS antisymmetric tensor B_{ab}, whose rank will be denoted by r.

As discussed in [11, 12], the quantised B_{ab} has a twofold effect on the Klein-bottle amplitude. The winding lattice now involves a projector, just like the transverse annulus amplitude of the toroidal model discussed in [10]. Moreover, as in [12] the Ω eigenvalues

of some of the twisted contributions are reverted, as demanded by the transverse channel amplitude, whose coefficients are to be perfect squares. Thus

$$\mathcal{K} = \tfrac{1}{4}(Q_o + Q_v)(0;0)\left[P_1 P_2 + 2^{-4}\sum_\epsilon W_1 W_2 e^{\frac{2i\pi}{\alpha'}n^{\mathrm{T}}B\epsilon}\right]$$
$$+\frac{2^{(4-r)/2}}{2}(Q_s + Q_c)(0;0)\left(\frac{\eta}{\vartheta_4(0)}\right)^2 . \tag{14}$$

Turning to the open sector, for the sake of brevity we shall again confine our attention to models without D5 branes, since the other cases can be easily reconstructed from these results. The quantised B_{ab} has a twofold effect on $\mathcal{A}$: it affects the momentum lattice and endows the contributions related to the Landau levels with additional multiplicities depending on the rank r of B_{ab}. Thus

$$\mathcal{A} = \tfrac{1}{4}\Bigg\{(Q_o + Q_v)(0;0)\bigg[(m+\bar{m})^2 2^{r-4}\sum_\epsilon P_1(B)P_2(B)$$
$$+2n\bar{n}2^{r-4}\sum_\epsilon \tilde{P}_1(B)\tilde{P}_2(B)\bigg]$$
$$-2\cdot 2^r(m+\bar{m})(n+\bar{n})(Q_o + Q_v)(z_1\tau;z_2\tau)\frac{k_1\eta}{\vartheta_1(z_1\tau)}\frac{k_2\eta}{\vartheta_1(z_2\tau)}$$
$$-2^r(n^2+\bar{n}^2)(Q_o + Q_v)(2z_1\tau;2z_2\tau)\frac{2k_1\eta}{\vartheta_1(2z_1\tau)}\frac{2k_2\eta}{\vartheta_1(2z_2\tau)}$$
$$-\left[(m-\bar{m})^2 - 2n\bar{n}\right](Q_o - Q_v)(0;0)\left(\frac{2\eta}{\vartheta_2(0)}\right)^2$$
$$-2(m-\bar{m})(n-\bar{n})(Q_o - Q_v)(z_1\tau;z_2\tau)\frac{2\eta}{\vartheta_2(z_1\tau)}\frac{2\eta}{\vartheta_2(z_2\tau)}$$
$$-(n^2+\bar{n}^2)(Q_o - Q_v)(2z_1\tau;2z_2\tau)\frac{2\eta}{\vartheta_2(2z_1\tau)}\frac{2\eta}{\vartheta_2(2z_2\tau)}\Bigg\} . \tag{15}$$

The Möbius amplitude can now be recovered, as usual, after a P transformation, from the transverse amplitudes $\tilde{\mathcal{K}}$ and $\tilde{\mathcal{A}}$, and reads

$$\mathcal{M} = -\tfrac{1}{4}\Bigg\{(m+\bar{m})(\hat{Q}_o + \hat{Q}_v)(0;0)2^{(r-4)/2}\sum_\epsilon \gamma_\epsilon P_1(B)P_2(B)$$
$$-(m+\bar{m})(\hat{Q}_o - \hat{Q}_v)(0;0)\left(\frac{2\hat{\eta}}{\hat{\vartheta}_2(0)}\right)^2$$
$$-2^{r/2}(n+\bar{n})(\hat{Q}_o + \hat{Q}_v)(2z_1\tau;2z_2\tau)\frac{2k_1\hat{\eta}}{\hat{\vartheta}_1(2z_1\tau)}\frac{2k_2\hat{\eta}}{\hat{\vartheta}_1(2z_2\tau)}$$
$$-(n+\bar{n})(\hat{Q}_o - \hat{Q}_v)(2z_1\tau;2z_2\tau)\frac{2\hat{\eta}}{\hat{\vartheta}_2(2z_1\tau)}\frac{2\hat{\eta}}{\hat{\vartheta}_2(2z_2\tau)}\Bigg\} , \tag{16}$$

where, as in [10, 12], the γ's are signs, required by the compatibility with the transverse channel, that determine the charge of the resulting O-planes.

The R-R tadpoles are modified, and become

$$m + \bar{m} + n + \bar{n} = 2^{5-r/2} \; ,$$
$$k_1 k_2 (n + \bar{n}) = 2^{5-r} \; , \tag{17}$$

so that the ranks of the gauge groups are reduced as usual, albeit here in an asymmetrical fashion.

We can now describe the massless spectrum of these magnetised orientifolds with a quantised NS-NS background. This clearly depends on the sign γ associated in $\mathcal{M}$ to the massless states, that determines the type of action (regular or projective) of the orbifold group on the CP group or, equivalently, the nature ("real" or "complex") of the CP multiplicities.

The more standard choice $\gamma_0 = +1$ requires a projective Z_2 action on the CP labels. Therefore, the resulting massless annulus and Möbius direct-channel amplitudes

$$\begin{aligned}
\mathcal{A}_0 \sim \tfrac{1}{4}\Big\{ & 4m\bar{m}Q_o(0) + 4n\bar{n}Q_o(0) + 2(m^2 + \bar{m}^2)Q_v(0) \\
& + (2 \cdot 2^r \cdot k_1 k_2 + 2 \cdot 4)(mn + \bar{m}\bar{n})Q_v(\zeta\tau) \\
& + (2 \cdot 2^r \cdot k_1 k_2 - 2 \cdot 4)(m\bar{n} + \bar{m}n)Q_v(\zeta\tau) \\
& + (4 \cdot 2^r \cdot k_1 k_2 + 4)(n^2 + \bar{n}^2)Q_v(\zeta\tau)\Big\} \; ,
\end{aligned} \tag{18}$$

and

$$\mathcal{M}_0 \sim -\tfrac{1}{2}(m + \bar{m})\hat{Q}_v(0) - \tfrac{1}{2}(2 \cdot 2^{r/2} \cdot k_1 k_2 + 2)(n + \bar{n})\hat{Q}_v(\zeta\tau) \; , \tag{19}$$

involve the "complex" multiplicities m and n.

Naively, these amplitudes would seem inconsistent: as a result of the further multiplicities related to the rank r of B_{ab}, *only some* of the string states with identical U(1) charges at their ends appear to contribute to $\mathcal{M}$. This is actually not the case, and the solution of the little puzzle follows a pattern that emerged from the study of SU(2) WZW models [24]. The multiplicities in the annulus count in general different, independent, sets of states, that are individually (anti)symmetrised by the Möbius amplitude, so that the corresponding coefficients in $\mathcal{A}$ and in $\mathcal{M}$ need only be equal modulo 2. The low-lying expansions

$$\begin{aligned}
Q_o(0) &\sim V_4 - 2C_4 \; ; & Q_v(0) &\sim 4O_4 - 2S_4 \; ; \\
Q_o(\zeta\tau) &\sim \text{massive} \; ; & Q_v(\zeta\tau) &\sim 2O_4 - S_4 \; ;
\end{aligned} \tag{20}$$

thus yield the massless spectrum

$$\begin{aligned}
(A + \overline{A}, 1) &+ \frac{2 \cdot 2^r \cdot k_1 k_2 + 2 \cdot 4}{4}(m, n) + \frac{2 \cdot 2^r \cdot k_1 k_2 - 2 \cdot 4}{4}(m, \bar{n}) \\
&+ \Big[k_1 k_2 \cdot (2^r + 2^{r/2}) + 2 \Big](1, A) + k_1 k_2 \cdot (2^r - 2^{r/2})(1, S) \; ,
\end{aligned} \tag{21}$$

r	(k_1, k_2)	n_T^c	n_H^c	G_{CP}	charged hypermultiplets
2	(1,1)	5	16	$\mathrm{U}(4) \times \mathrm{U}(4)$	$(6 + \overline{6}, 1) + 4\,(4, 4)$
					$+ 8\,(1, 6) + 2\,(1, 10)$
2	(1,2)	5	16	$\mathrm{U}(6) \times \mathrm{U}(2)$	$(15 + \overline{15}, 1_0) + 6\,(6, 2_+)$
					$+ 2\,(6, 2_-) + 14\,(1, 1_{++}) + 4\,(1, 3)$
2	(2,2)	5	16	$\mathrm{U}(7) \times \mathrm{U}(1)$	$(21 + \overline{21}, 1_0) + 10\,(7, 1_+)$
					$+ 6\,(7, 1_-) + 8\,(1, 1_{++})$
4	(1,1)	7	14	$\mathrm{U}(3) \times \mathrm{U}(1)$	$(3 + \overline{3}, 1) + 10\,(3, 1_+)$
					$+ 6\,(3, 1_-) + 12\,(1, 1_{++})$

Table 1: Massless spectra for $\gamma_0 = +1$.

with gauge group $\mathrm{U}(m) \times \mathrm{U}(n)$, where $S(A)$ denotes the corresponding (anti)-symmetric representation.

Altogether, the tadpole equations admit four inequivalent solutions, and the corresponding spectra (aside from the universal $\mathcal{N} = (1, 0)$ gravity multiplet) are summarised in table 1.

As in the non-magnetised case, the choice $\gamma_\epsilon = -1$ in the Möbius amplitude induces a regular action of the Z_2 orbifold on the CP charges [12]. The corresponding multiplicities, now "real", require also a different embedding of the magnetic $\mathrm{U}(1)$'s, so that

$$
\begin{aligned}
m + n + \bar{m} + \bar{n} &\rightarrow m_1 + n + \bar{n} + m_2 \ , \\
m + n - \bar{m} - \bar{n} &\rightarrow m_1 + n + \bar{n} - m_2 \ ,
\end{aligned}
\tag{22}
$$

and the direct-channel annulus and Möbius massless contributions become

$$
\begin{aligned}
\mathcal{A}_0 \ \sim \ & \tfrac{1}{2}(m_1^2 + m_2^2)Q_o(0) + n\bar{n}Q_o(0) + m_1 m_2 Q_v(0) \\
& + \Big\{ \tfrac{1}{4}\left[2 \cdot 2^r \cdot k_1 k_2 - 2 \cdot 4\right] m_1(n + \bar{n}) \\
& + \tfrac{1}{4}\left[2 \cdot 2^r \cdot k_1 k_2 + 2 \cdot 4\right] m_2(n + \bar{n}) \Big\} Q_v(\zeta\tau) \\
& + \tfrac{1}{2}\left[2 \cdot 2^r \cdot k_1 k_2 - 2\right](n^2 + \bar{n}^2)Q_v(\zeta\tau) \ ,
\end{aligned}
\tag{23}
$$

and

$$
\mathcal{M}_0 \sim -\tfrac{1}{2}\Big\{ -(m_1 + m_2)\hat{Q}_o(0) + \left[2 \cdot 2^{r/2} \cdot k_1 k_2 + 2\right](n + \bar{n})\hat{Q}_v(\zeta\tau) \Big\} \ .
\tag{24}
$$

For these models with real CP charges, the untwisted tadpole conditions

$$
\begin{aligned}
m_1 + m_2 + n + \bar{n} &= 2^{5 - r/2} \ , \\
k_1 k_2(n + \bar{n}) &= 2^{5 - r} \ ,
\end{aligned}
\tag{25}
$$

have to to be supplemented by the twisted tadpole condition

$$m_1 + n + \bar{n} = m_2 \ . \tag{26}$$

A possible solution with $r = 2$ and $k_1 = k_2 = 1$ is $m_1 = 0$, $m_2 = 8$ and $n = 4$, and yields a massless spectrum with a gauge group $\mathrm{USp}(8) \times \mathrm{U}(4)$ comprising, aside from the $\mathcal{N} = (1,0)$ gravity multiplet, 5 tensor multiplets, 16 neutral hypermultiplets, and additional charged hypermultiplets in the representations $4(8,4) + 6(1,6)$. As in conventional tori [10] and orbifolds [12], a continuous Wilson line can actually connect these two classes of magnetised vacua.

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